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COMMENT

The resistive susceptibility of a three-dimensional random diode–insulator network

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Abstract. The resistive susceptibility $\chi_R(p)$ of Harris and Fisch is expanded graphically for the directed simple cubic bond problem as a power series in the diode density p up to p^{11}. Standard Padé analysis shows that $\chi_R(p)$ diverges with the exponent $\gamma_R = 2.70 \pm 0.04$ and using a scaling relation the exponent t for the conductivity of the infinite cluster is estimated to be $t = 1.29 \pm 0.02$.

In a previous letter (Bhatti and Essam 1984), the critical exponent for the resistive susceptibility of a random mixture of diode and insulators on the square lattice was calculated. The resulting value of the conductivity exponent t was in good agreement with the Monte Carlo calculation of Arora *et al* (1983), but slightly inconsistent with that of Redner and Muller (1982). So far there have been no estimates of t for three-dimensional lattices and here we extend our two-dimensional work to bond percolation on the simple cubic lattice. A first estimate of the critical probability p_c, for this problem was obtained by Blease (1977) using the first eight terms of the mean size series $S(p)$. Recently (De'Bell and Essam 1983) this series was extended by five terms using the transfer matrix method with the result $p_c = 0.382 \pm 0.001$. In our previous calculation the transfer matrix method was found to be much less effective for the resistive susceptibility and instead the non-nodal graph technique was used. It was found that the resistive susceptibility for any acyclically directed lattice is given by

$$\chi_R(p) = \Psi_R(p)[S(p)]^2$$

where $\Psi_R(p)$ is the contribution to $\chi_R(p)$ from non-nodal graphs. For the simple cubic lattice we have listed all such graphs with up to eleven edges (there are 28 of these). The resulting rational coefficients of $\Psi_R(p)$ up to p^{11} are shown in table 1. Using the known series for $S(p)$ (Blease 1977, De'Bell and Essam 1983) we have calculated $\chi_R(p)$ and the coefficients rounded to 26 digits are given in table 2. During the computation of the resistance of the subgraphs used to determine the weights in the graph expansion no negative flows occurred.

The Padé analysis of the $\chi_R(p)$ series gives rise to the pole–residue plot shown in figure 1, from which we estimate that $\chi_R(p)$ diverges with critical exponent $\gamma_R = 2.698 + 36\Delta p_c \pm 0.004$. Here Δp_c is the deviation from the assumed central value of

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$p_c = 0.382$. Since the uncertainty in the p_c makes the largest contribution to the error in γ_R we consider that extension of the $\chi_R(p)$ series to the length obtained by De'Bell and Essam (1983) for $S(p)$ would not significantly improve our final estimate for γ_R.

Table 1. $\Psi_R(p) = 3p - 9p^4 + \Sigma_{n=6}^{\infty} a_n p^n$.

n	a_n(Num)	a_n(Den)
6	−135	2
7	294	5
8	−2388	5
9	85 901	115
10	−30 305 193	8855
11	54 556 528 845 842	7526 103 585

Table 2. $\chi_R(p) = \Sigma_{n=1}^{\infty} b_n p^n$.

n	b_n
1	0.300 000 000 000 000 000 000 000 00D+01
2	0.180 000 000 000 000 000 000 000 00D+02
3	0.810 000 000 000 000 000 000 000 00D+02
4	0.315 000 000 000 000 000 000 000 00D+03
5	0.114 300 000 000 000 000 000 000 00D+04
6	0.390 150 000 000 000 000 000 000 00D+04
7	0.129 288 000 000 000 000 000 000 00D+05
8	0.413 517 000 000 000 000 000 000 00D+05
9	0.130 220 965 217 391 304 347 826 09D+06
10	0.400 521 109 260 304 912 478 825 52D+06
11	0.122 106 014 255 705 836 129 546 18D+07

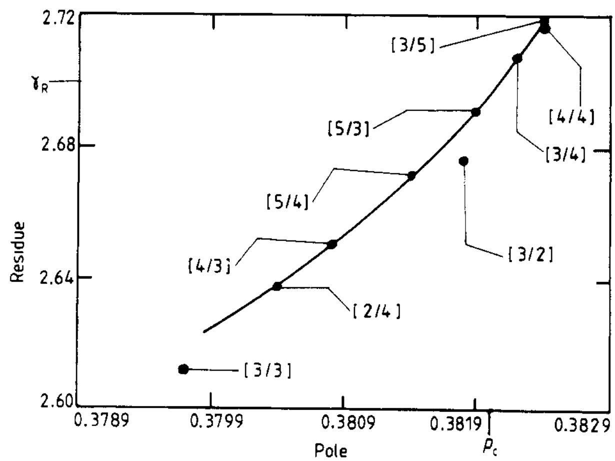

Figure 1. Pole–residue plot for Padé approximants to d log $\chi_R(p)$.

Using the scaling formula for the conductivity exponent of the diode problem obtained by Redner (1982)

$$t = \zeta_R + (d-1)\nu_\perp - \nu_\parallel$$

with $\zeta_R = \gamma_R - \gamma$, we obtain $t = 1.292 \pm 2\Delta p_c \pm 0.016$. The values of γ, $\nu_\perp$, $\nu_\parallel$ used in the above calculation were obtained by De'Bell and Essam (1983). The small coefficient of Δp_c arises from the cancellation of the systematic errors. Using the result $|\Delta p_c| \leqslant 0.001$ of De'Bell and Essam (1983) we obtain the biased estimates of γ_R and t quoted in the abstract.

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